

Generation of Fock states and qubits in periodically pulsed nonlinear oscillator

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We demonstrate a quantum regime of dissipative nonlinear oscillators where the creation of Fock states as well as the superpositions of Fock states are realized for time-intervals exceeding the characteristic decoherence time. The preparation of quantum states is conditioned by strong Kerr nonlinearity as well as by excitation of resolved lower oscillatory energy levels with a specific train of Gaussian pulses. This provides practical signatures to look for in experiments with cooled nonlinear oscillators.

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I. INTRODUCTION

The preparation and use of Fock states, that have definite numbers of energy quanta, and various superpositions of Fock states form the basis of quantum computation and communications [1]. Quantum dynamics of an oscillator is naturally described by Fock states, however, these states are hard to create in experiments. The reason is that excitations of oscillatory systems usually lead to the production of thermal states or coherent states but not quantum Fock states. Nevertheless, quantum oscillatory states can be prepared and can be manipulated by coupling oscillators to atomic systems. In this way, a classical pulse applied to the atomic states creates a quantum state that can subsequently be transferred to the harmonic oscillator excited in a coherent state. The systematic procedure has been proposed in Ref. [2] and has been demonstrated for deterministic preparation of mechanical oscillatory Fock states with trapped ions [3] and in cavity QEDs with Rydberg atoms [4]. Most recently the analogous procedure has been applied in solid-state circuit QED for deterministic preparation of photon number states in a resonator by interposing a highly nonlinear Josephson phase qubit between a superconducting resonator [5].

In this paper, we show that the production of Fock states and their various two-state superpositions or qubits can also be realized for an anharmonic oscillator without any interactions with atomic and spin-1/2 systems. For this goal we consider nonlinear dissipative oscillator the (NDO) under sequence of classical Gaussian pulses separated by time intervals. The nonlinearity effects in the NDO should be enough to break the equidistant of oscillatory energy levels. In this case, the oscillatory energy levels are well resolved, and selection is possible near the resonant excitation for them. It is demonstrated that the creation of oscillatory nonclassical

states can be realized in a periodically pulsed oscillator with complete consideration of decoherence effects and in a regime of strong Kerr nonlinearity.

It is well assessed, that in a free decoherence regime an anharmonic oscillator leads to Schrödinger cat states which are destroyed due to dissipation and decoherence. This situation of vanishing quantum superposition states of the NDO in an over transient dissipative operational regime has been strongly demonstrated within the framework of the Wigner function that visualizes quantum interference as negative values in the phase space. Really surprisingly analytical results for the Wigner functions of the NDO under monochromatic excitation have been analytically obtained [6], [7] in the steady-state regime that are positive in all ranges of the phase space. This Wigner function mainly displays only conventional attributes of the model such as bistability or turning points. In the extreme quantum regime of the system corresponding to strong nonlinearities or low damping, the Wigner function displays a rich variety of phase transition images in the bistable operation regime, qualitatively different from what one would expect from the corresponding semiclassical analysis but not squeezing and the other nonclassical effects.

Here, we demonstrate that in the specific pulsed regime of the NDO the production of the Fock states are realized as well as the superposition of the Fock states. These states can be created for time intervals exceeding the characteristic time of decoherence. The corresponding Wigner functions of the oscillatory mode show ranges of negative values and gradually deviate from the Wigner function of an NDO driven by monochromatic driving.

Previously, a driven NDO operated in a quantum regime has become more important in both fundamental and applied sciences, particularly, for the implementation of basic quantum optical systems in the engineering of nonclassical states and quantum logic. In these systems, the efficiency of the quantum effects requires a high nonlinearity with respect to dissipation. On the other hand, ground state cooling is critical for reaching quantum regimes of oscillatory systems. In this regard, the largest Kerr nonlinearities were proposed for many physical systems: cavity quantum electrodynamics-based de-

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vices [8], [9], in electromagnetically induced transparency [10], and for cooling nanoelectromechanical systems and nano-opto-mechanical systems based on various oscillators [11],[12]. Superconducting devices based on the non-linearity of the Josephson junction (JJ) that exhibits a wide variety of quantum phenomena [13]-[19] offer an unprecedented high level of nonlinearity and low quantum noise. In some of these devices dynamics is analogous to those of a quantum particle in an oscillatory anharmonic potential [20], [21] and behaves like artificial atoms [22]. Note that the comparison of second-order nonlinearities taking place for various quantum devices has been recently analyzed in Ref. [23] and the NDO for the case of strong nonlinearity has been considered in [24], [25].

The paper is arranged as follows. In Sec. II, we describe a pulsed NDO and consider some physical implementations of the model. In Sec. III, we shortly discuss the NDO under a monochromatic driving for both transient and steady-state regimes. In Sec. IV, we consider the production of Fock states in the pulsed regime of the NDO. In Sec. V, we present the results for the construction of quantum superposition of Fock states on the base of the Rabi oscillations and the Wigner functions. We summarize our results in Sec. VI.

II. MODEL DESCRIPTION AND IMPLEMENTATIONS

The Hamiltonian of an anharmonic driven oscillator in the rotating wave approximation reads as:

$$H = \hbar\Delta a^\dagger a + \hbar\chi(a^\dagger a)^2 + \hbar f(t)(\Omega a^\dagger + \Omega^* a), \quad (1)$$

where time dependent coupling constant $\Omega f(t)$, that is proportional to the amplitude of the driving field, consists of the Gaussian pulses with duration T , which are separated by time intervals τ ,

$$f(t) = \sum e^{-(t-t_0-n\tau)^2/T^2}. \quad (2)$$

Here, a^\dagger , a are the oscillatory creation and annihilation operators, χ is the nonlinearity strength, and $\Delta = \omega_0 - \omega$ is the detuning between the mean frequency of the driving field and the frequency of the oscillator.

This model seems experimentally feasible and can be realized in several physical systems, particularly, for the production of Fock states. In fact, the effective Hamiltonian (1) describes a nanomechanical oscillator with a^\dagger and a raising and lowering operators related to the position and momentum operators of a mode quantum motion,

$$x = \sqrt{\frac{\hbar}{2m\omega_0}}(a + a^\dagger), \quad p = -i\sqrt{2\hbar m\omega_0}(a - a^\dagger), \quad (3)$$

where m is the effective mass of the nanomechanical resonator, ω_0 is the linear resonator frequency, and χ is proportional to the Duffing nonlinearity. It is possible

to reach the quantum regime for such frequencies, i.e., to cool down the temperatures for which thermal energy is comparable to the energy of oscillatory quanta. Recent progress in cooling mechanical oscillators to their ground states has been achieved in Refs. [26]-[31], (see also Ref. [32] and the references therein). This interest is caused by measuring the quantum effects [33]-[36] and by preparing resonators in states with high purity to exploit their quantum behavior in future technologies [37], [38].

One of the variants of nano-oscillators is based on a double-clamped platinum beam [39] for which the non-linearity parameter equals $\chi = \hbar/4\sqrt{3}Qma_c^2$, where a_c is the critical amplitude at which the resonance amplitude has an infinite slope as a function of the driving frequency, and Q is the mechanical quality factor of the resonator. In this case, the giant nonlinearity with the strength $\chi \cong 3.4 \cdot 10^{-4} s^{-1}$ was realized. Note, that details of this resonator, including the expression for the parameter a_c , are presented in Ref. [40].

It is known that a single light mode propagated in Kerr media with dielectric constant ε and $\chi^{(3)}$ susceptibility is described by the effective Hamiltonian (1). In this case the parameter χ reads as $\chi = \frac{9}{8} \frac{\hbar\omega^2\chi^{(3)}}{\varepsilon^2 S d}$, where S is the cross-sectional area of the beam and d is the envelope width [41],[42].

The same Hamiltonian describes a current-biased Josephson junction in the rotating wave approximation. In this case, creation and annihilation operators are expressed in the following terms:

$$a = \frac{1}{\sqrt{2}} \left\{ \left(\frac{E_J}{E_C} \right)^{1/4} \phi + i \left(\frac{E_C}{E_J} \right)^{1/4} n \right\}, \quad (4)$$

$$a^\dagger = \frac{1}{\sqrt{2}} \left\{ \left(\frac{E_J}{E_C} \right)^{1/4} \phi - i \left(\frac{E_C}{E_J} \right)^{1/4} n \right\}, \quad (5)$$

where n is the number operator of Cooper-pair charges, ϕ is the phase difference between superconducting elements in the Josephson junction, E_J is the junction coupling energy, and E_C is the electrostatic Coulomb energy for a single Cooper pair. The parameter of non-linearity and driving amplitude for this case reads as $\chi = -\frac{E_C}{48}$, $\Omega = -\frac{I\Phi_0}{8\sqrt{2}\pi\eta^{3/2}}$, where I is the biasing current and $\Phi_0 = h/2e$ is the superconducting flux quantum $\eta = \sqrt{E_J/E_C}$. The typical values for the $|0\rangle \rightarrow |1\rangle$ transition frequency in the Josephson junction used in the experiments cooled to $20mK$ are $\sim 10GHz$ and $\sim 10\mu A$ for biasing currents. The possible decay path $\gamma/2\pi$ is on the order of 2.6 MHz, and hence, χ/γ is 6.9 [43], [44]. Thus, the current-biased JJ combines negligible dissipation with extremely large nonlinearity. However, a large nonlinearity in the Josephson inductance is obtained by biasing the junction at a current I that is very close to the critical current. As nonlinearity of the Josephson inductance breaks the degeneracy of the oscillatory energy level spacings, the dynamics of states in near to resonant excitation is close to the two-level one.

We have included dissipation and decoherence in the NDO on the basis of the master equation,

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \sum_{i=1,2} \left(L_i \rho L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \rho - \frac{1}{2} \rho L_i^\dagger L_i \right), \quad (6)$$

where $L_1 = \sqrt{(N+1)\gamma}a$ and $L_2 = \sqrt{N\gamma}a^\dagger$ are the Lindblad operators, γ is a dissipation rate, and N denotes the mean number of quanta of a heat bath. To study the pure quantum effects we focus on the cases of very low reservoir temperatures which, however, ought to be still larger than the characteristic temperature $T \gg T_{cr} = \hbar\gamma/k_B$. This restriction implies that dissipative effects can be described self-consistently in the frame of the Lindblad Eq. (6). For clarity, in our numerical calculation we choose the mean number of reservoir photons $N = 0$. Note, that for $N \ll 1$ the above mentioned restriction is valid for the majority of problems of quantum optics and, particularly, for the schemes involving the nano-mechanical oscillator and Josephson junction. In experiments, the nonlinear oscillator based on the current-biased JJ is cooled down to $T = 20mK$ which corresponds to $N = 0.0013$, whereas $T_{cr} = 10^{-5}$ K, for $\gamma = 1MHz$.

The time evolution of a NDO driven by a coherent force cannot be solved analytically for arbitrary evolution times and suitable numerical methods have to be used. Nevertheless, a dissipative driven nonlinear oscillator has been solved exactly (that means consideration of all order of dissipation) in the steady-state regime and in terms of the Fokker-Planck equation in complex P representation [7], [41]. An analogous solution has been obtained for a driven parametric oscillator combined with Kerr nonlinearity [6]. The Wigner functions for both these models have been obtained using these solutions [6], [7]. The investigation of quantum dynamics of a driven dissipative nonlinear oscillator for nonstationary cases, particularly, for various pulsed regimes, is much more complicated and only a few papers have been written in this field up to now.

Quantum effects in a NDO with a time-modulated driving force have been studied in a series of papers [24], [45]-[47] in the context of a quantum stochastic resonance [45], quantum dissipative chaos [46], [47], and quantum interference assisted by a bistability [24]. The dynamics of the periodically driven nonlinear oscillator also has been studied in the Refs. [48]-[51]. The application of quantum scissors [52] to a periodically driven cavity with a Kerr medium has been considered in Ref. [53] for the truncation of a coherent state to a superposition of a vacuum and single-photon Fock state.

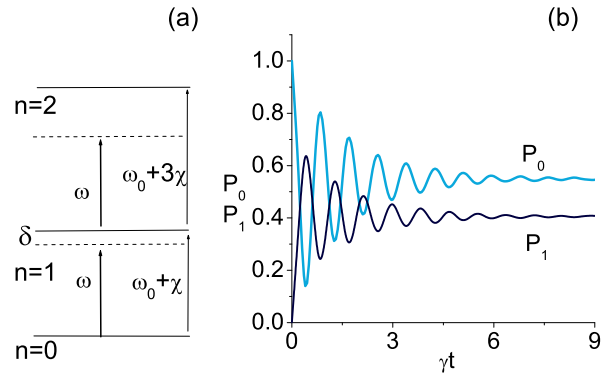


FIG. 1. (Color online) (a) Energy levels of anharmonic oscillator and (b) Rabi oscillations of the state populations with decoherence which suppresses beating. The parameters are as follows: $\Delta/\gamma = -11$, $\chi/\gamma = 15$, and $\Omega/\gamma = 7$

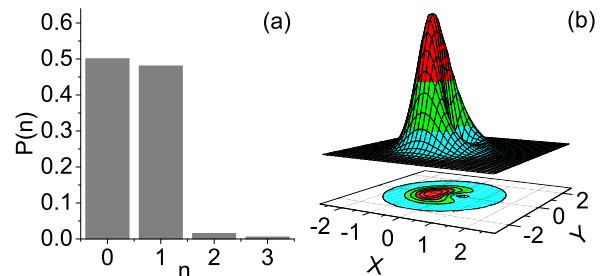


FIG. 2. (Color online) (a) The excitation numbers distribution and (b) the Wigner function for the NDO in steady-state. The parameters are as in Fig. 1.

III. MONOCHROMATICALLY DRIVEN NDO: RABI OSCILLATIONS AND WIGNER FUNCTIONS

In the absence of any driving, the quantized vibration states of the nonlinear oscillator are the Fock states $|n\rangle$, which are spaced in energy $E_n = E_0 + \hbar\omega_0 n + \hbar\chi n^2$ with $n = 0, 1, \dots$. The levels form an anharmonic ladder [see Fig. 1(a)] with anharmonicity that is given by $E_{21} - E_{10} = 2\hbar\chi$. In the case of monochromatic excitation the energetic spectrum is also shifted and in the second-order of perturbation, the theory with the interaction part $\hbar(\Omega a^\dagger + \Omega^* a)$ of the Hamiltonian described by Eq. (1) reads as $\Delta E_n = \hbar\Omega^2 \left(\frac{n}{\omega + \chi(2n-1)} - \frac{(n+1)}{\omega + \chi(2n+1)} \right)$.

Below we concentrate on quantum regimes for the parameters when oscillatory energy levels are well resolved considering near to resonant transitions between lower number states $|0\rangle \rightarrow |n\rangle$. We solve the master equation Eq. (6) numerically based on the quantum state diffusion method [55]. The applications of this method for studies

of the NDO can be found in Refs. [45]–[47]. In the calculations, a finite basis of number states $|n\rangle$ is kept large enough (where n_{max} is typically 50) so that the highest energy states are never populated appreciably. In the following the distribution of oscillatory excitation states $P(n) = \langle n|\rho|n\rangle$ as well as the Wigner functions,

$$W(r, \theta) = \sum_{n,m} \rho_{nm}(t) W_{mn}(r, \theta) \quad (7)$$

in terms of the matrix elements $\rho_{nm} = \langle n|\rho|m\rangle$ of the density operator in the Fock state representation will be calculated. Here (r, θ) are the polar coordinates in the complex phase-space plane, $x = r \cos \theta$, $y = r \sin \theta$, whereas, the coefficients $W_{mn}(r, \theta)$ are the Fourier transform of matrix elements of the Wigner characteristic function.

We start with a discussion of the main idea by considering a first stage NDO under monochromatic excitation, [$f(t) = 1$ in the Hamiltonian described by Eq. (1)]. It is well known that this system exhibits regions of regular and bistable motions with the control parameters χ, Δ , and Ω . Below, we consider the monostable operational regime of a pulsed NDO that is realized for negative values of the detuning satisfying $\chi(\Delta + \chi) \geq 0$ [41].

The results for the lowest excitation $|0\rangle \rightarrow |1\rangle$ are depicted in Fig. 1 for the parameters Ω/γ and the detuning $\delta = \frac{1}{\hbar}(E_1 - E_0) - \omega = \Delta + \chi$ meeting the near to resonant condition, $\delta = 4\gamma$. In this case, only two levels effectively are involved in the Rabi-like oscillations of the populations P_0 and P_1 of the vacuum and first excitation number state $|1\rangle$ that is demonstrated in Fig. 1 (b). These oscillations take place on the Rabi frequency $R = \sqrt{\Omega^2 + \delta^2}$ that characterizes a two-level quantum system in a monochromatic field, whereas decay of the amplitude takes place due to dissipation. Here, we have assumed that the initial oscillatory state is a vacuum state.

Analyzing a monochromatically driven NDO in an over transient regime for time intervals, $t \gg \gamma^{-1}$, we use both a numerical method and the analytical results obtained in terms of the exact solution of the Fokker-Planck equation [6], [7]. The exact analytical results for a dissipative-driven nonlinear oscillator have been obtained in terms of the Fokker-Planck equation in the complex P-representation. In this way, the solution for the Wigner function of the oscillatory mode involving quantum noise in all order of perturbation theory and in the steady-state regime have been derived

$$W(\alpha) = N \exp^{-2|\alpha|^2} \left| \frac{J_{\lambda-1}(\sqrt{-8\alpha\varepsilon})}{(\alpha^*)^{(\lambda-1)/2}} \right|^2. \quad (8)$$

Here, J_λ is the Bessel function, $\lambda = (\gamma + i\Delta)/\chi$, $\varepsilon = \Omega/\chi$, and amplitude $\alpha = x + iy$ is the complex C-number variable corresponding to the operator a . It is evident from this result that, under monochromatic force, the NDO does not display any quantum interference pattern

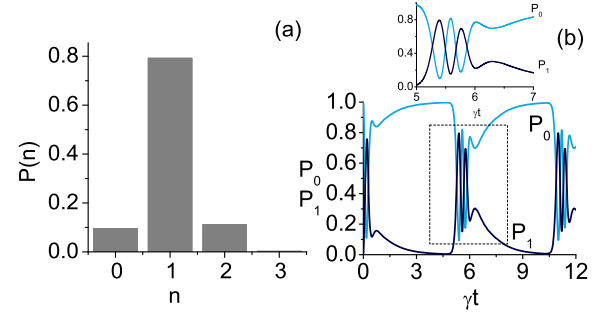


FIG. 3. (Color online) The excitation number distribution for time intervals corresponding to the maximal population (a) of the $|1\rangle$ state and (b) time-dependent populations. The inset shows the enlarged graphic to visualize Rabi-type oscillations in detail. The parameters are as follows: $\Delta/\gamma = -15$, $\chi/\gamma = 15$, $\Omega/\gamma = 6$, $\tau = 5.5\gamma^{-1}$, and $T = 0.4\gamma^{-1}$.

and cannot generate superposition states as the corresponding Wigner function is positive in all phase space. It should be noted that the steady-state solution of the Fokker-Planck equation has been found using the approximation method of potential equations [41], [54]. The validity of this solution has not been checked up to now in the strong quantum regime that requires a high nonlinearity with respect to dissipation. For this reason, we also calculate the Wigner function numerically on the basis of the numerical simulation of the master equation by using the quantum state diffusion method [55]. According to these calculations, the system reaches the equilibrium of oscillatory states in the steady-state regime and the Wigner function of the oscillatory mode is positive in all phase-space describing statistical mixtures of states $|0\rangle$ and $|1\rangle$. The result is displayed in Fig. 2 for the parameters that have been used in the Fig. 1.

IV. PRODUCTION OF FOCK STATES IN THE PULSED REGIME

Now, we are able to present the main results of this paper concerning production of Fock states and qubits for the NDO due to pulsed excitation. In this sense we note the main peculiarity of our paper. We investigate the production of nonclassical states for interaction time intervals exceeding the characteristic time of the dissipative processes, $t \gg \gamma^{-1}$, however, in the nonstationary regime that is conditioned by the specific form of excitation.

In this way, we consider two important regimes leading to the production of both Fock states and superposition of Fock states for the low-power resonance transition $|0\rangle \rightarrow |1\rangle$ between vibrational states. Each of these regimes is realized for the appropriate choosing parameters, the detuning Δ , the Rabi amplitude R , and the parameters of pulses. For the pulsed NDO, the ensemble-

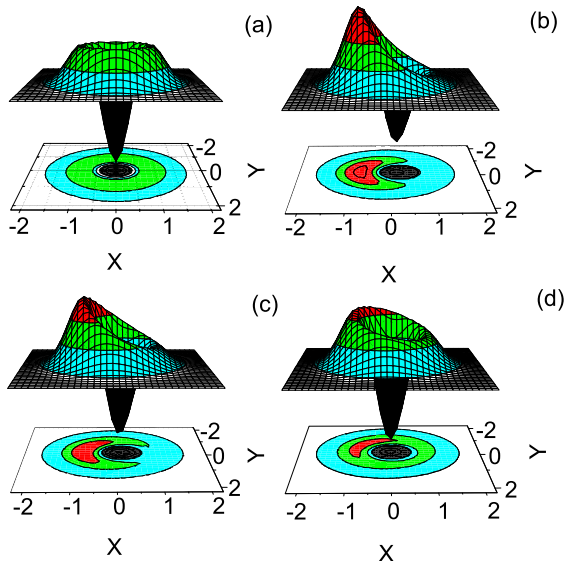


FIG. 4. (Color online) (a) The Wigner function for pure $|1\rangle$ state. Evolution of the Wigner function for (b) $t = 3\tau - 0.5T$, (c) $t = 3\tau - 0.4T$, and (d) $t = 3\tau - 0.25T$ for the parameters: $\Delta/\gamma = -15$, $\chi/\gamma = 15$, $\Omega/\gamma = 6$, $T = 0.4\gamma^{-1}$, and $\tau = 5.5\gamma^{-1}$. The ranges of negativity are indicated in black.

averaged mean oscillatory excitation numbers, the populations of oscillatory states and the Wigner functions are nonstationary and exhibit a periodic time dependent behavior, i.e., repeat the periodicity of the pump laser in an over transient regime.

We demonstrate below that Fock state $|1\rangle$ can be effectively produced if the pulsed excitation is tuned to the exact resonance $\delta = \Delta + \chi = 0$. The other condition concerns the controlling parameters of the pulses. We assume the regime involving short pulses separated by long time intervals that allows increasing the weight of a single Fock state. In these cases, the probability distribution of excitation number $P(n)$ displays a maximum approximately equal to unity for the one-photon state $n = 1$. The condition of low excitation reads as $|\Delta/\Omega| > 1$. It allows avoiding mixed states, as the external field populates both $|n + 1\rangle$ and $|n - 1\rangle$ states. The typical results for pulse duration $T = 0.4\gamma^{-1}$ and the time interval between pulses $\tau = 5.5\gamma^{-1}$ are shown in Fig. 3. The time evolution of the probabilities P_0 and P_1 of the $|0\rangle$ and $|1\rangle$ states starting from the vacuum state are depicted in Fig. 3(b). As we see, the maximal weight 0.8 of P_1 is realized for the definite time-intervals of measurement $t = k\tau - 0.25T$, $k = 1, 2, 3, \dots$, [see Fig. 3(a)].

Note that, due to the anharmonicity, this system is close to the two-level limit for small amplitudes, and the resonant excitation and the time evolution are essentially quantum. As is well known, in the case of a res-

onant excitation of a two-level atomic system by a laser pulse with the electric field envelope $E(t)$, the population of the excited state oscillates as $\sin^2(\frac{1}{2}R(t))$, where $R(t) = \frac{1}{\hbar} \int_{-\infty}^t dt' (E(t')d)$ is the time dependent Rabi frequency, and d is the dipole moment of atomic transition. For the resonant excitation of the nonlinear oscillator under a single pulse, the Rabi frequency is determined by the time-dependent coupling constant $\Omega(t)$ (see also Ref. [20]). In this case, the Rabi-like oscillations have been observed for the Josephson device driven by resonant microwave flux pulses that behaves as an anharmonic oscillator [20]. The corresponding Wigner functions that show negative values have been also calculated in decoherence free approximation for time intervals during the Rabi period. If the NDO is driven by a sequence of pulses, this behavior is modified and essentially depends on the duration of the pulses as well as from time intervals between them. As our calculations show, in this case, Rabi-like oscillations take place for short time intervals between two adjacent pulses. The detailed description of such oscillations is depicted in Fig. 3(b), the inset graphic. Analyzing time evaluation of the Wigner functions during pulses we conclude the production of Fock state $|1\rangle$ in the pulsed NDO. The results of the calculations of the Wigner function are presented in Fig. 4 for various time intervals within the pulse duration and are compared with the Wigner function of the pure state $|1\rangle$ [see Fig. 4(a)]. It is easy to realize that the Wigner function depicted in Fig. 4(d) for time interval $t = 3\tau - 0.5T$ corresponding to the maximal population P_1 displays a ring signature with the center at $x = y = 0$ in the phase-space that is a characterized pure Fock state and, hence, is closest to the pure state $|1\rangle$ Wigner function [see Fig. 4(a)]. An intuitive explanation of this point has been performed above on the basis of the results depicted in Fig. 3.

V. QUANTUM SUPERPOSITION OF FOCK STATES

The most striking signature of the periodically pulsed NDO is the appearance of quantum interference between Fock states in the over transient regime. We demonstrate it for near-to-resonant transition $|0\rangle \rightarrow |1\rangle$ with the oscillatory parameters used above in Figs. 1 and 2. The typical results for the dynamics of the excitation numbers and the Wigner function for the pulse duration $T = 0.7\gamma^{-1}$, and the time interval between pulses $\tau = 2.2\gamma^{-1}$ are demonstrated in Figs. 5 and 6.

A qualitative explanation for the production of Fock superposition states can be obtained by considering the time evolution of the anharmonic oscillator in the over-transient regime, where dissipation and decoherence effects are essential. This analysis allows us to choose the time intervals for the effective joint excitation of states $|0\rangle$ and $|1\rangle$. In this way, time evolution of the mean excitation number and the populations of vacuum and

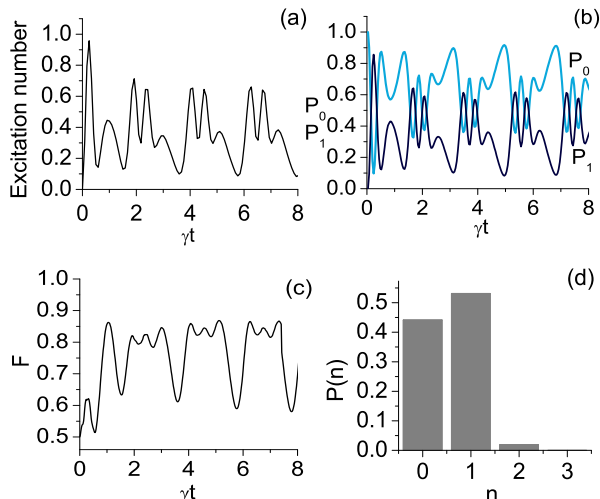


FIG. 5. (Color online) (a) The mean excitation number, (b) populations of oscillatory states, (c) time evaluation of the fidelity, and (d) the distribution of excitation number for time interval $t = 5\tau - 0.4T$. The parameters are as follows: $\Delta/\gamma = -11$, $\chi/\gamma = 15$, $\Omega/\gamma = 7$, $T = 0.7\gamma^{-1}$, and $\tau = 2.2\gamma^{-1}$.

excited states are calculated and are depicted in Figs. 5(a) and 5(b). As we see, for the overtransient regime, the time modulation of the averaged excitation number and the populations of two oscillatory states repeat the periodicity of the pump laser. The populations display time dependent Rabi-like oscillations in the ranges of time intervals between two adjacent pulses that offer a clear signature of quantum behavior and the transitions between the lowest eigenstates. The existence of a Rabi-like oscillation under the influence of the driving, which is resonant with transitions between neighboring levels, is closely linked to the anharmonicity of the oscillator so that oscillations do not show up in a harmonic oscillator driven by resonant monochromatic excitation in the overtransient regime. Moreover, it is intuitively clear that quantum interferences and the forming of Fock superposition states are possible in the course of these Rabi-like oscillations that involve the crossing of two populations.

To demonstrate this statement, we investigate the Wigner function for the parameters using in Fig. 5. Using these data, in Fig. 6, we plot the evolution of the Wigner function that demonstrates interference fringes on the phase space between $|0\rangle$ and $|1\rangle$ states. Analyzing this evolution, we conclude that the closest to the pure superposition state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, the Wigner function [see, Fig. 6(a)] is realized for the measurement time intervals $t = k\tau - 0.5T$, $k = 1, 2, 3, \dots$ within the regime of Rabi-like oscillations for which the populations of the two lower states are approximately equal [see Figs. 5(d)]. This conclusion is also supported by the calculation of the time-dependent fidelity $F = |\langle\Psi|\rho|\Psi\rangle|$ depicted on Fig. 5(c). As we see, the time-dependence of the fidelity re-

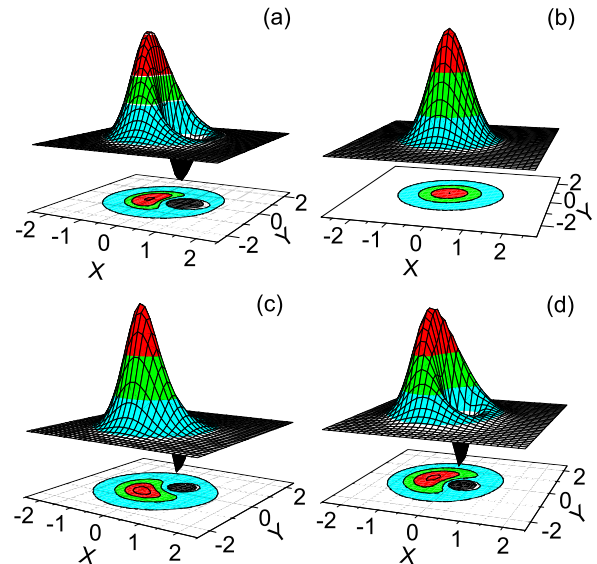


FIG. 6. (Color online) (a) The Wigner function for the pure $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ state. The Wigner function evaluation for the parameters used in Fig. 5: (b) $t = 5\tau - 1.4T$, (c) $t = 5\tau - 0.9T$, and (d) $t = 5\tau - 0.4T$. The ranges of negativity are indicated in black.

peats the evolution of the pulses and reaches the maximal values 0.88 for time intervals $t = k\tau - 0.5T$, $k = 1, 2, 3, \dots$, corresponding to the Wigner function in Fig. 6(d). It is important that the maximal fidelity is realized when the oscillatory mean excitation number reaches its maximal value, thus quantum interference effects between $|0\rangle$ and $|1\rangle$ states are strong in the case of a high power excitation. We realize that production of quantum interference in the overtransient regime is due to the control of the decoherence through the application of suitable tailored, synchronized pulses [see, for example, Refs. [56], [57]]. Indeed, quantum interference is realized here if a mutual influence of pulses is essential (for $\tau/T = 3.14$ in Fig. 6). In this case the decay of Rabi-like oscillations is negligible for time intervals between the pulses that is reflected in the production of quantum interference for the dissipative regime.

VI. CONCLUSION

We have shown that the production of the Fock states as well as the superposition of the Fock states are possible in the single-mode dissipative oscillator due to strong Kerr nonlinearities and the periodically pulsed excitation. The nonlinearities should be strong enough to avoid the degeneracy of the oscillatory energy-level spacings as well as to reach quantum operational regimes, that are realized for $\chi/\gamma > 1$. Quantum interference between

Fock states in the overtransient regime is realized due to driving the oscillator by a series of short pulses with proper parameters for effective reducing of the dissipative and decoherence effects. These results have been demonstrated by numerical calculations of the dynamics of oscillatory excitation numbers and the Wigner functions of oscillatory mode for the various over transient regimes of NDO. We hope that this approach can be expanded to include the production of the other lower Fock states $|2\rangle$, $|3\rangle$, etc.

This problem of engineering various quantum states has attracted increasing interest in recent years not only

for quantum information processing, but also for various other quantum technologies based on nano-mechanical oscillators, particularly, in optomechanics, for the manipulation of mechanical degrees of freedom by light scattering. We believe that the scheme under consideration, operated with a train of realistic Gaussian pulses, is available in these areas for experiments with laser or microwave pulses [see, for example Refs. [20] and [32]]. Together with the recent advancements in the engineering of various cooled oscillatory systems, these results seem to be important for the further studies of quantum phenomena in light of new technologies.

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